

IMPROVEMENT OF CONTROL PERFORMANCES USING FRACTIONAL $PI^\lambda D^\mu$ CONTROLLERS

K. Bettou., A. Charef

*Département d'Electronique
Université Mentouri Route Ain El-bey-25000 - Constantine Algérie
E-mail: bettou_kh@yahoo.com*

Abstract: This paper deals with the tuning of fractional $PI^\lambda D^\mu$ controllers, in which the orders of the integral and derivative parts, λ and μ , respectively, are fractional. The purpose is to take advantage of the introduction of these two parameters and fulfil additional specifications of design. A new method for tuning the fractional $PI^\lambda D^\mu$ controller is proposed in this paper. The basic ideas of this new tuning method are based, in the first place, on the classical Ziegler-Nichols tuning method for setting the parameters of the fractional $PI^\lambda D^\mu$ controller for $\lambda=\mu=1$ which means setting the parameters of the classical PID controller, and on the minimum ISE criterion by using the Hall-Sartorius method for setting the fractional integration action order λ and the fractional differentiation action order μ .

Illustrative examples and simulation results are presented to show the control quality enhancement of this proposed fractional $PI^\lambda D^\mu$ controller conception method compared to the PID controller conception using the Ziegler-Nichols tuning method. It is shown that the fractional $PI^\lambda D^\mu$ controller gives more robustness and good control behaviour in presence of noise.

Keywords: PID Tuning, Fractional $PI^\lambda D^\mu$, ISE Optimization, Control System Design, Industrial Process Control.

1. Introduction

The PID controllers have remained, by far, as the most commonly used in practically all industrial feedback control applications. The main reason is its relatively simple structure, which can be easily understood and implemented in practice. They are thus more acceptable than advanced controllers in practical applications unless evidence shows that they are insufficient to meet specifications. Many techniques have been suggested for their parameters tuning (1), (2) since Ziegler and Nichols published their parameter tuning method in 1942 (3). Although all the existing techniques for the PID controller parameter tuning, a continuous and an intensive research work is still underway towards system control quality enhancement and performance improvements.

On the other hand, in recent years it is remarkable the increasing number of studies related with the application of fractional controllers in many areas of science and engineering. This fact is due to a better understanding of the fractional calculus potentialities.

Many applications of fractional order differentiation can be found in engineering (4), (5). In the field of automatic control, the fractional order controllers which are the generalization of classical integer order controllers would lead to more precise and robust control performances. Although it is reasonably true, as also argued in (6), that the fractional order models require the fractional order controllers to achieve the best performance, but in most cases the researchers consider the fractional order controllers applied to regular linear or nonlinear dynamics to enhance the system control performances.

Historically there are four major types of fractional order controllers (6), (7):

- CRONE Controller
- Tilted Proportional and Integral (TID) Controller
- Fractional Order $PI^\lambda D^\mu$ Controller
- Fractional Lead-Lag Compensator

However, fractional order PI^λD^μ controller is the most distinguished controller among them. Meanwhile, many researchers have been trying to find a proper method to design and tune a fractional order PI^λD^μ controller. Some of these methods are based on, an extension of classical PID control theory. To this respect in (8) the extension of derivation order from integer to fractional numbers provides a more flexible tuning strategy and therefore an easier achieving of control requirements with respect to classical controllers. In (9) an optimal fractional order PI^λD^μ controller based on specified gain margin and phase margin with a minimum ISE criterion has been designed by using a differential evolution algorithm. An experimental investigation has been presented in (10) where the fractional order PI^λD^μ controller has been applied for active reduction of vertical tail buffeting. A fractional order control strategy has also been successfully applied in the control of a power electronic buck converter (11), more concretely a fractional sliding mode control. In this paper we propose the design of the fractional order PI^λD^μ controller of a classical unity feedback control system. And the controller is the fractional order PI^λD^μ controller whose transfer function is given as:

$$C(s) = K_p \left(1 + \frac{1}{T_I s^\lambda} + T_D s^\mu \right) \quad (1)$$

With K_P is the proportional constant, T_I is the integration constant, T_D is the differentiation constant, λ is the fractional integration action order such that 0 < λ < 2 and μ is the fractional differentiation action order such that 0 < μ < 2.

The interest of this kind of controllers is justified by a better flexibility, since it has two more parameters which are the fractional integration action order λ and the fractional differentiation action order μ. These parameters can be used to fulfill additional specifications for the design or other interesting requirements for the controlled system.

However, if the controller is designed using the form defined by Eq.1, it is obvious that its time-domain simulation or implementation will require to band-limit its fractional effect. Low and high frequency band-limitations avoid the use of an infinite number of rational modes to approximate the fractional parts, and furthermore, the high-frequency band-limitation of the derivative effect limits its high-frequency gain and thus the control effort provided by the controller.

This paper is organized as follows. Section 2 introduces the fractional order calculus. In section 3, we introduce the rational approximation function of the fractional PI^λD^μ controller. In section 4, basic ideas and the derived formulations of the new conception strategy of the fractional PI^λD^μ controller are presented. In section 5, some illustrative examples are presented to demonstrate the advantages of the proposed tuning method. Section 6 draws the main conclusions.

2. A Brief Introduction to Fractional Order Calculus

A commonly used definition of fractional differo-integral is the Riemann-Liouville definition

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{d}{dt} \right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} d\tau \quad (2)$$

For $m-1 < \alpha < m$; where, $\Gamma(\cdot)$ is the well-known Euler's gamma function. An alternative definition, based on the concept of fractional differentiation, is Grunwald-Letnikov definition given by

$${}_a D_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{\Gamma(\alpha) h^\alpha} \sum_{k=0}^{(t-a)/h} \frac{\Gamma(\alpha+k)}{\Gamma(k+1)} f(t-kh) \quad (3)$$

One can observe that by introducing the notion of fractional order operator ${}_a D_t^\alpha f(t)$ the differentiator and integrator can unified.

Another useful tool is the Laplace transform. The Laplace transform of an n -th derivative ($n \in R_+$) of a signal $x(t)$ relaxed at $t=0$ is given by: $L\{D^n x(t)\} = s^n X(s)$. So, a fractional order differential equation, provided both the signal $u(t)$ and $y(t)$ are relaxed at $t=0$, can be expressed in a transfer function form

$$G(s) = \frac{a_1 s^{\alpha_1} + a_2 s^{\alpha_2} + \dots + a_{m_A} s^{\alpha_{m_A}}}{b_1 s^{\beta_1} + b_2 s^{\beta_2} + \dots + b_{m_B} s^{\beta_{m_B}}} \quad (4)$$

Where $(a_m, b_m) \in R^2, (\alpha_m, \beta_m) \in R_+^2, \forall (m \in N)$

Thorough expositions of these subjects may be found in (12) and (13).

3. Rational Function of $PI^\lambda D^\mu$ Controller

3.1 Fractional Order $PI^\lambda D^\mu$ Controller

The most common form of a fractional order controller is the $PI^\lambda D^\mu$ controller (4), involving an integrator of order λ and a differentiator of order μ where λ and μ can be any real numbers. The transfer function of such a controller has the following form

$$C(s) = K_p \left(1 + \frac{1}{T_i s^\lambda} + T_D s^\mu \right) \quad (5)$$

The equation for the fractional order $PI^\lambda D^\mu$ controller's output in time domain is:

$$u(t) = K_p \left(e(t) + \frac{1}{T_i} .D^{-\lambda} e(t) + T_D .D^\mu e(t) \right) \quad (6)$$

Clearly, selecting $\lambda=1$ and $\mu=1$, a classical PID controller can be recovered. Using $\lambda=1$ and $\mu=0$, and $\lambda=0$ and $\mu=1$, respectively, corresponds to the conventional PI and PD controllers. All these classical types of PID controllers are special cases of the fractional order $PI^\lambda D^\mu$ controller given by Eq.5 it can be expected that the fractional order $PI^\lambda D^\mu$ controller may enhance the systems control performances. One of the most important advantages of the fractional order $PI^\lambda D^\mu$ controller is the possible better control of fractional order dynamical systems. Another advantages lies in the fact that the fractional order $PI^\lambda D^\mu$ controllers are less sensitive to changes of parameters of a controlled system (4). This is due to the two extra degrees of freedom to better adjust the dynamical properties of a fractional order control system.

3.2 Fractional Order Integrator

The integration action of the fractional order PID controller is a fractional order integrator which is represented in the frequency domain by the following irrational function:

$$C_i(s) = \frac{1}{s^\lambda} \quad (7)$$

With λ is a positive real number such that $0 < \lambda < 1$. in a given frequency band of practical interest $[\omega_L, \omega_H]$, the fractional order integrator can be modeled by a fractional power pole (FPP) whose transfer function is given as follows:

$$C_i(s) = \frac{K_i}{\left(1 + \frac{s}{\omega_c} \right)^\lambda} \quad (8)$$

The FPP of Eq.5 and consequently the fractional order integrator of Eq.4 is approximated by a rational function in the frequency band $[\omega_L, \omega_H]$ as (15):

$$C_I(s) = \frac{K_I}{\left(1 + \frac{s}{\omega_c}\right)} = K_I \frac{\prod_{i=0}^{N-1} \left(1 + \frac{s}{z_i}\right)}{\prod_{i=0}^N \left(1 + \frac{s}{p_i}\right)} \quad (9)$$

The poles p_i 's and the zeros z_i 's are given as in (14).

3.3 Fractional Order Differentiator

The transfer function of the differentiation action of the fractional $PI^\lambda D^\mu$ controller is a fractional order differentiator which is represented in the frequency domain by the following irrational function:

$$C_I(s) = s^\mu \quad (10)$$

With μ is a positive real number such that $0 < \mu < 1$. In a given frequency band of practical interest $[\omega_L, \omega_H]$, the fractional order differentiator can be modeled by a fractional power zero (FPZ) whose transfer function is given as follows:

$$C_I(s) = \left(1 + \frac{s}{\omega_c}\right)^\mu \quad (11)$$

The FPZ of Eq.8 and consequently the fractional order integrator of Eq.7 is approximated by a rational function in the frequency band $[\omega_L, \omega_H]$ as (15):

$$C_I(s) = \left(1 + \frac{s}{\omega_c}\right)^\mu = K_D \frac{\prod_{i=0}^N \left(1 + \frac{s}{z_D}\right)}{\prod_{i=0}^N \left(1 + \frac{s}{p_D}\right)} \quad (12)$$

The poles p_d 's and the zeros z_d 's are given as in (14).

3.4 Fractional $PI^\lambda D^\mu$ Controller

In sections 3.2 and 3.3, we showed how we can approximate the fractional order integrator and differentiator by rational functions, for a given frequency band of practical interest $[\omega_L, \omega_H]$; so Eq.1 becomes:

$$C(s) = K_p \left(1 + \frac{K_I}{T_I} \frac{\prod_{i=0}^{N_I-1} \left(1 + \frac{s}{z_{I_i}}\right)}{\prod_{i=0}^{N_I} \left(1 + \frac{s}{p_{I_i}}\right)} + T_D K_D \frac{\prod_{i=0}^{N_D} \left(1 + \frac{s}{z_{D_i}}\right)}{\prod_{i=0}^{N_D} \left(1 + \frac{s}{p_{D_i}}\right)} \right) \quad (13)$$

The poles p_{I_i} 's, the zeros z_{I_i} 's, and the parameters K_I and N_I of the rational function approximation of the fractional order integrator, and the zeros z_{D_i} 's, the poles p_{D_i} 's, and the parameters K_D and N_D of the rational function approximation of the fractional order differentiator can be easily calculated from (14), (15).

4. Fractional $PI^\lambda D^\mu$ Controller Design

4.1 Design of the Parameters K_p , T_I and T_D

Our tuning strategy is based, in the first place, on the classical Ziegler-Nichols tuning rules for setting the parameters K_p , T_I and T_D of the fractional $PI^\lambda D^\mu$ controller for $\lambda = \mu = 1$ which means setting the parameters of a simple conventional PID controller.

4.2 Hall-Sartorius Method

The Hall-Sartorius method (16), consist of finding, for a linear system, a controller minimizing the integral square error (ISE) of a classical unity feedback control system for a unit step input. The integral square error (ISE) is given as:

$$J = \int_0^{\infty} [e(t)]^2 dt = \int_0^{\infty} [r(t) - y(t)]^2 dt \quad (14)$$

Where $e(t)$ is the error signal. According to Laplace transform properties the integral J can be written as (16):

$$J = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} E(s)E(-s)ds \quad (15)$$

Then, for $E(s)$ a rational function in s given as:

$$E(s) = \frac{N_E(s)}{D_E(s)} \quad (16)$$

The complex integral J will be:

$$J = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} \frac{N_E(s)N_E(-s)}{D_E(s)D_E(-s)} ds \quad (17)$$

The complex integral of Eq.14 is obtained as in (16):

$$J = \frac{(-1)^{(n-1)}}{2} \frac{\det(\Delta_N)}{\det(\Delta_D)} \quad (18)$$

4.3 Design of the Fractional Order Parameters

With the parameters K_P , T_I and T_D obtained in the first step, we use the Hall-Sartorius method (16), to determine the optimum settings of the fractional integration order λ and the fractional differentiation action order μ of the fractional $PI^\lambda D^\mu$ controller. In other words, our proposed method consists using the parameters K_P , T_I and T_D for setting the parameters λ and μ minimizing the integral square error (ISE) of the classical unity feedback control system of Fig.1 for a unit step input.

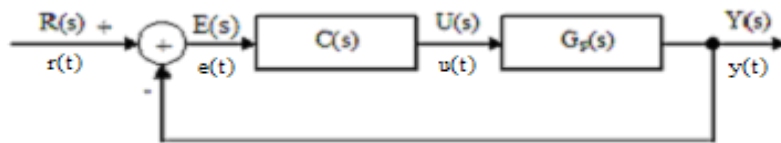


Fig. 1. Classical unity feedback control system.

From Eq.17, the complex integral $J(\lambda, \mu)$ is given as:

$$J(\lambda, \mu) = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} E(s)E(-s)ds \quad (19)$$

From Fig.1, the error signal $E(s)$ is given as:

$$E(s) = \left(\frac{1}{1 + C(s)G_p(s)} \right) R(s) = \left(\frac{1}{1 + C(s)G_p(s)} \right) \left(\frac{1}{s} \right) \quad (20)$$

5. Simulation Results

In the following two different popular chemical control processes are used to evaluate the control performance of the proposed controller scheme. In each example, the performance of the proposed controller is compared to the performance of conventional PID controller, which widely used to control the chemical processes. To find the conventional PID parameters, we have used Ziegler-Nichols tuning method.

5.1 Tank Level Control

Despite the model is nonlinear (as the flow rate out of the tank depends on the square root of the level), a first order plus time delay model has been estimated by applying the area method to open-loop response with a step from 2V to 2.5V at the process input (17). The first order plus time delay model obtained is

$$G_p = \frac{1.98}{(1+29s)} e^{-11s} \quad (23)$$

For $\lambda=1$, $\mu=1$ and using the Ziegler-Nichols tuning method, the controller's parameters K_p , T_i and T_D are found to be $K_p=1.597$, $T_i=22$, $T_D=5.5$. Hence, the conventional PID controller's transfer function $C_1(s)$ is given as:

$$C_1(s) = 1.597 \left(1 + \frac{1}{22s} + 5.5s \right) \quad (24)$$

Using the parameters K_p , T_i and T_D found above, the fractional $PI^\lambda D^\mu$ controller's transfer function $C_2(s)$ is:

$$C_2(s) = 1.597 \left(1 + \frac{1}{22s^\lambda} + 5.5s^\mu \right) \quad (25)$$

The smallest ISE index J of section 4 is obtained for the couple $(\lambda, \mu) = (0.9, 1.1)$. Then the fractional $PI^\lambda D^\mu$ controller's transfer function $C_2(s)$ required is given as:

$$C_2(s) = 1.597 \left(1 + \frac{1}{22s^{0.9}} + 5.5s^{1.1} \right) \quad (26)$$

A band-limit implementation of fractional order $PI^\lambda D^\mu$ controller is important in practice, and the finite dimensional approximation of the fractional order $PI^\lambda D^\mu$ controller should be in a proper range of frequencies of practical interest.

In this example of application, the fractional order integral and derivative parts have been implemented by CHAREF approximation of the fractional integrator and differentiator (14) and (15), choosing a frequency band from $0.01\omega_c$ to $100\omega_c$, with ω_c is the unity gain crossover frequency of the open loop transfer function $C(s)G_p(s)$ when $C(s)$ is the conventional PID controller tuned by Ziegler-Nichols method.

Figure 2 shows the step responses of the closed loop system with both controllers.

There could be variation or uncertainty in the process parameters. If the gain or the time constant of the process are changed:

$$G_p = \frac{1.98}{(1+29s)} e^{-11s} \quad (27)$$

$$G_p = \frac{1.98}{(1+29s)} e^{-11s} \quad (28)$$

Then the step responses of the closed loop system with both controllers are given in Fig.3 and fig.4 respectively.

From the results obtained in the case of study, it can be concluded that the compensated system using the proposed fractional order $PI^{0.9}D^{1.1}$ controller is robust to changes in static gain and time constant, since variations of the performance characteristics are lower for the fractional order $PI^{0.9}D^{1.1}$ controller.

In short, it can be said that the use of the fractional order $PI^\lambda D^\mu$ controller provide better responses and robust system.

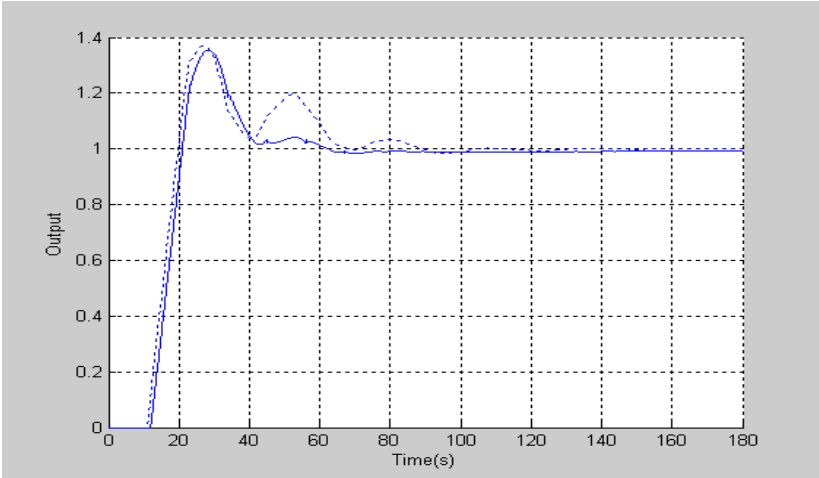


Fig.2. Step responses of the closed loop system with $C(s)$ a conventional PID controller (dashed line) and $C(s)$ a fractional $PI^{0.9}D^{1.1}$ controller (solid line)

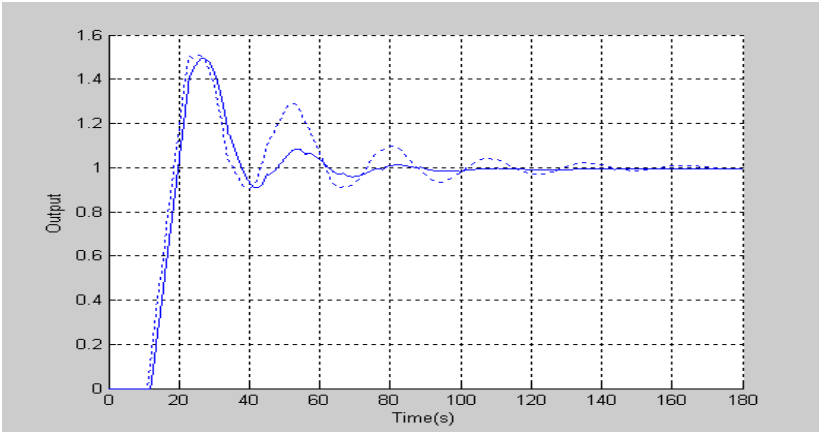


Fig.3. Step response of the closed loop system $C(s)$ the PID controller $C_1(s)$ (Dotted line) and $C(s)$ the proposed $PI^{0.9}D^{1.1}$ controller $C_2(s)$ (solid line), with static gain variation: 2.2.

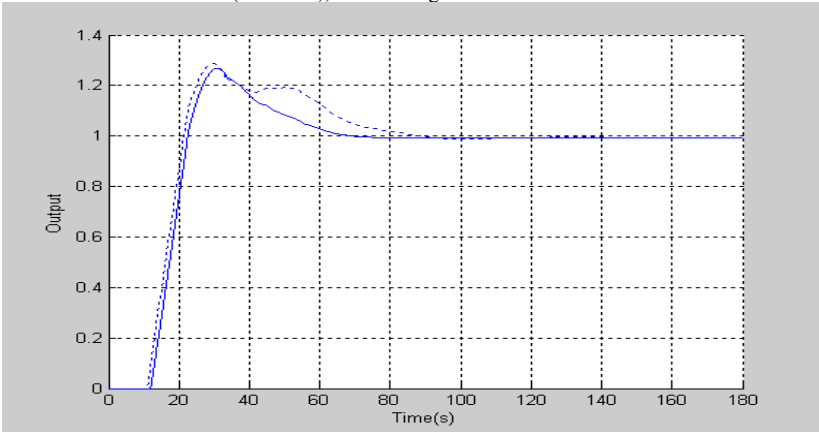


Fig. 4. Step response of the closed loop system $C(s)$ the PID controller $C_1(s)$ (Dotted line) and $C(s)$ the proposed $PI^{0.9}D^{1.1}$ controller $C_2(s)$ (solid line), with time constant variation: 35.

5.2 Tank Temperature Control

The next example is the temperature control in a tank (18), which has a typical continuous time transfer function, given by

$$G_p = \frac{0.41}{s(1+50s)} e^{-50s} \quad (29)$$

The manipulated variable in this case is the duty cycle of the heating resistor. The process has integral effect and was identified around the nominal operating condition (50°C). The delay time of this process is also common in the most chemical processes and cause ordinary controllers to fail in many cases.

For $\lambda=1$, $\mu=1$ and using the Ziegler-Nichols tuning method, the controller's parameters K_p , T_I and T_D are found to be $K_p = 0.0489$, $T_I = 125$, $T_D = 31.25$. Hence, the conventional PID controller's transfer function $C_1(s)$ is given as:

$$C_1(s) = 0.0489 \left(1 + \frac{1}{125s} + 31.25s \right) \quad (30)$$

Using the parameters K_p , T_I and T_D found above, the fractional $PI^\lambda D^\mu$ controller's transfer function $C_2(s)$ is:

$$C_2(s) = 0.0489 \left(1 + \frac{1}{125s^\lambda} + 31.25s^\mu \right) \quad (31)$$

The smallest ISE index J of section (2) is obtained for the couple $(\lambda, \mu) = (0.4, 0.8)$. Then the fractional $PI^\lambda D^\mu$ controller's transfer function $C_2(s)$ required is given as:

$$C_2(s) = 0.0489 \left(1 + \frac{1}{125s^{0.4}} + 31.25s^{0.8} \right) \quad (32)$$

Figure 5, shows the step responses of the closed loop system with both controllers.

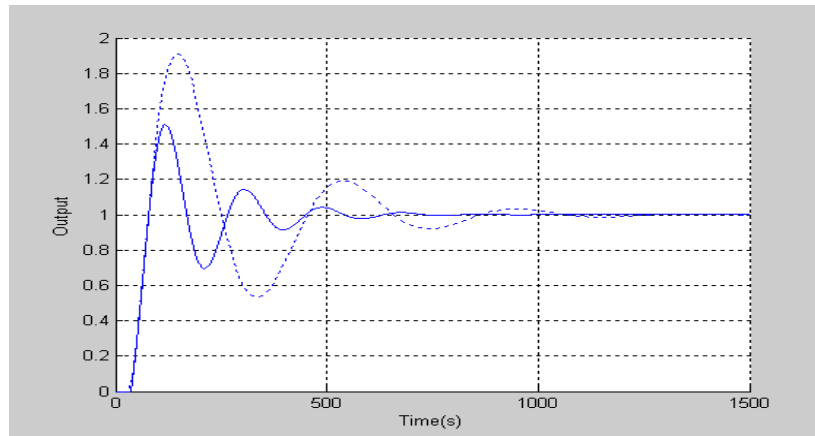


Fig.5. Step responses of the closed loop system with $C(s)$ a conventional PID controller (dashed line) and $C(s)$ a fractional $PI^{0.4}D^{0.8}$ controller (solid line)

There could be variation or uncertainty in the process parameters. If the time constant of the process is changed:

$$G_p = \frac{0.41}{s(1+75s)} e^{-50s} \quad (33)$$

In order to test to test the fractional $PI^\lambda D^\mu$ controller in the presence of noise, the above process is tested for two different variances $\sigma^2=0.0005$ and $\sigma^2=0.001$. the closed loop responses for different variances white noise are illustrated in Fig.7 and Fig.8. As it can be seen the fractional $PI^\lambda D^\mu$ controller shows very good noise rejection feature. Even in the presence of very high noise, the system is able to trace the desired response.

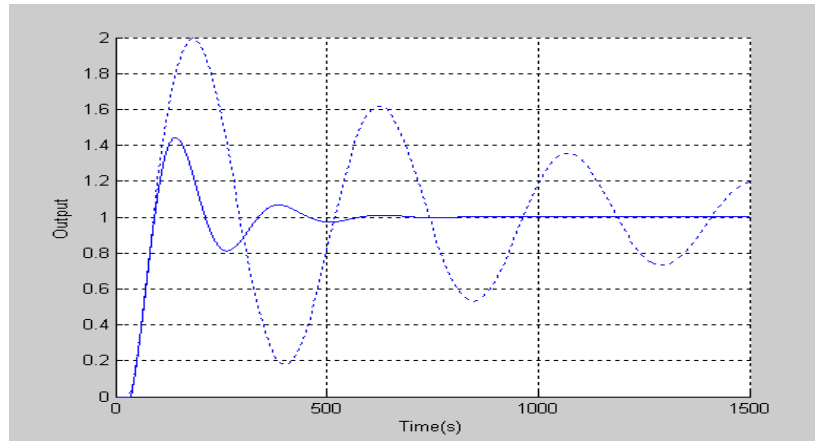


Fig. 6. Step response of the closed loop system $C(s)$ the PID controller $C_1(s)$ (Dotted line) and $C(s)$ the proposed $PI^{\lambda}D^{\mu}$ controller $C_2(s)$ (solid line), with time constant variation: 75.

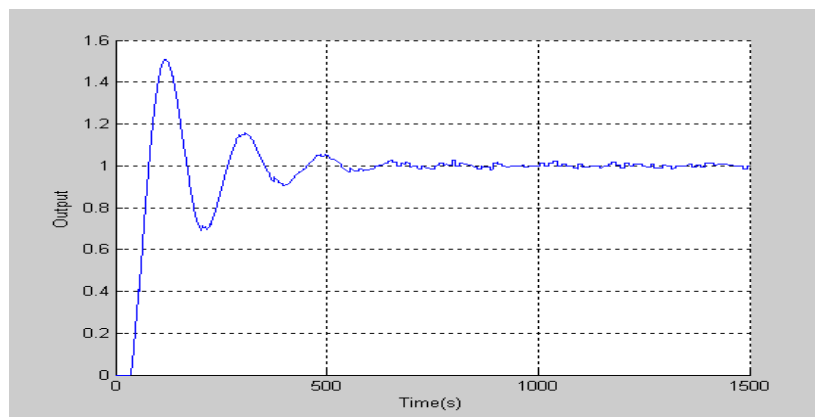


Fig. 7. Step response of the closed loop system for $\sigma_2=0.0005$

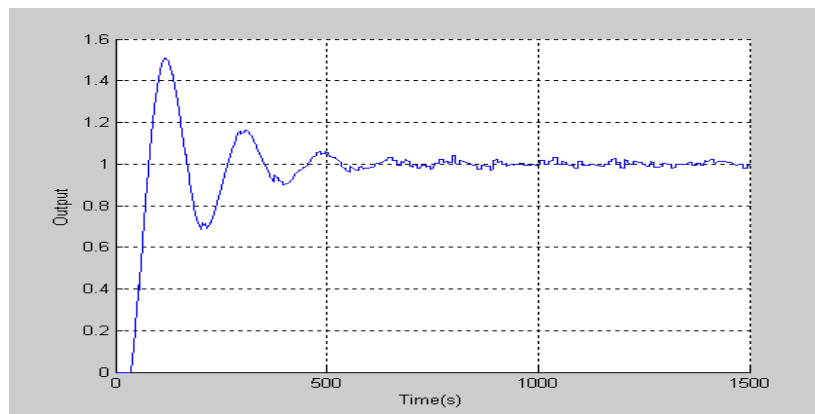


Fig. 8. Step response of the closed loop system for $\sigma_2=0.001$

6. Conclusion

In this study, a new fractional $PI^{\lambda}D^{\mu}$ controller tuning process based on Ziegler-Nichols tuning method and ISE algorithm was developed.

The performance of this tuning method for the fractional $PI^{\lambda}D^{\mu}$ controller tuning was tested in different situations (varying the steady state gain, and changing a pole of the system). The fractional $PI^{\lambda}D^{\mu}$ controller showed high robustness in all cases compared to the conventional PID controller. The fractional $PI^{\lambda}D^{\mu}$ controller is applied to a model with noise. Even in the presence of very high noise, the fractional $PI^{\lambda}D^{\mu}$ controller showed good control behaviour. System is perturbed in several ways and having noise rejection even if very high noise variance exists.

Reference

- 1- K. Åström, T. Hägglund, "PID Controllers: Theory, Design and Tuning," Instrument Society of America, North Carolina, 1995.
- 2- Y.Cheng-Ching, "Autotuning of PID Controllers Advances in Industrial Control," Springer-Verlag, 1999.
- 3- J.G. Ziegler, and N.B. Nichols, "Optimum Settings for Automatic Controllers," *A.S.M.E Trans.* Vol. 64, N° 8, pp. 759-768, 1942.
- 4- I. Podlubny, "Fractional order systems and $PI^{\lambda}D^{\mu}$ controllers," *IEEE Trans. Automatic Control*, Vol. 44, N°1, pp. 208-214, 1999.
- 5- N. Engheta, 'On Fractional Calculus and Fractional Multipoles in Electromagnetism', *IEEE Trans. Antenna Propagat*, Vol.51, pp.470-483, 2003.
- 6- Y.Q.Chen, D,Xue, and H,Dou, « Fractional Calculus and Biomimetic Control », *IEEE Int. Conf. on robotics and biomimetics (ROBIO 04)*, Shengang, China, Aug 2004.
- 7- Y.Q.chen, D,Xue, 'A Comparative Introduction of Four Fractional Order Controllers,' *In Proc. Of the 4th IEEE World Congress on Intelligent Control and Automatic (WCICA02)*, pp. 3228-3235, Shanghai, China, Jun 2002.
- 8- R. Caponetto, L, Fortuna and D, Porto 'Parameter Tuning of a Non Integer Order PID Controller' *In Proc. The Fifteenth International Symposium on Mathematical Theory of Networks and Systems*. Notre Dame, Indiana.2002.
- 9- Leu. J., Tsay and C. Hwang, 'Design Of Optimal Fractional-Order Controller'. *Journal of The Chinese Institute of Chemical Engineers* 33(2), 193-202. 2002.
- 10- Sanchez, . 'Fractional PID Control for Active Reduction of Vertical Tail Buffeting .' Master's Thesis. Saint Louis University, USA.1999.
- 11- Calderon, A, J.,B,M, Vinagre and V, Feliu , 'Fractional Sliding Mode Control of a DC-DC Buck Converter with Application to DC Motor Drives. *In: ICAR 2003: The 11th Inter. Conf. on Advanced Robotics*. Coimbra, Portugal, pp. 252-257. '2003
- 12- I. Podlubny, 'Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations to Method of Their Solution and Some of their Applications' Academic Pres, San Diego, 1999.
- 13- K. Miller, B. Ross, 'an Introduction to the Fractional Calculus and Fractional Differential Equations, Wiley, New York; 1993.
- 14- Charef, A.*et al.* 'Fractal System as Represented by Singularity Function'. *IEEE Tran. on Auto. Cont.* Vol. 37, N°9, pp. 1465-1470. 1992.
- 15- Charef, A., 'Analogue Realization of Fractional Order Integrator, Differentiator and Fractional $PI^{\lambda}D^{\mu}$ Controllers'. *IEE Proc. on Control Theory Applications*. Vol.135, N°6, pp. 714-720. 2006.
- 16- Borne P. *et al.*, 'Analyse et Régulation des Processus Industriels', Tome 1 : Régulation Continue, Editions Technip, Paris, 1993.
- 17- Aurlio .P, Antonio V., 'A Noncausal Approach for the Improvement of PID Control Performances'. *Proc. of American control conference*. Boston, Massachusetts, 2004.
- 18- Carlos, B., Eduardo, F. Camacho, 'A Generalized Predictive Controller for a Wide Class of Industrial Processes', *IEEE Trans. on Control Systems Technology*, Vol.6, No.3, 1998.