IMPROVEMENT OF CONTROL PERFORMANCES USING FRACTIONAL ORDER PROPORTIONAL INTEGRAL PROPORTIONAL DERIVATIVE (PI^{α}-PD^{β}) CONTROLLERS

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ABSTRACT: In this paper we propose a fractional PI^{α}-PD^{β} controller that tuned with integral performance criterion. The orders of the integral and derivative parts, α and β, respectively, are fractional. This controller is a generalization of a conventional PI-PD controller. This expansion could provide much more flexibility than a conventional PI-PD controller design. The tuning method is based on the solution of an optimization problem. For setting the six parameters of the fractional PI^{α}-PD^{β} controller, the proposed method is based on the minimum integral squared error (ISE) criterion with a minimum control effort. The integral criterion is calculated by using Hall-Sartaurius method. Two cases of study are given; the results demonstrate that gains and orders optimization leads to better transient performance of the proposed fractional PI^{α}-PD^{β} control structure.

Keywords: Controllers Tuning, Hall-Sartautius method, Integral Criterion, PI-PD Controllers, PI^{α}-PD^{β} control

1. INTRODUCTION

Despite the significant developments of recent years in control theory and technology, Proportional-Integral-Derivative (PID) controller is the most industrially used control algorithm. This can be explained by its simplicity, low cost and ability to solve most of control problems [1]. Difficulties are often faced, however, while controlling plants with resonance, integral or unstable transfer functions [2].

PI-PD controller proposed by S. Majhi and D.P. Atherton [3] is a modified form of PID controller. The PI-PD controller, which corresponds to PI control of the plant transfer function changed by the PD feedback, can produce improved control in several situations[4].

In the last decades, there are growing numbers of applications of fractional calculus in different areas of control engineering [5-7]**.** This fact is due to a better understanding of the fractional calculus potentialities. For example, in [8] the authors have showed that the fractional $PI^{\lambda}D^{\mu}$ controllers provide a more flexible tuning strategy that can achieve control requirements which can never be realized with classical PID controllers. In [9] a design method of the fractional $PI^{\lambda}D^{\mu}$ controller is derived based on specified gain and phase margins with a minimum integral squared error (ISE) criterion. A proposition for the implementation of a fractional PI^{λ} controller for first-order systems with long time delay has been given in [10]. In [11] the objective of the design technique is to find out optimum settings for a fractional $PI^{\lambda}D^{\mu}$ controller in order to fulfill five different design specifications for the closed-loop system. In [12] the objective of the design of the fractional $PI^{\lambda}D^{\mu}$ controller is such that the feedback control system fulfills different specifications.

This paper is organized as follows. Section 2 we discuss the proposed fractional order PI^{α}-PD^{β} control structure, also we introduce the approximation of the fractional PI^{α}-PD^{β} controller by a rational function in a limited frequency band of interest, the method of Hall-Sartorius used for criterion calculus is presented. In section 3, we introduce the basic ideas and the derived formulations of the new conception strategy of the fractional PI^{α}-PD^{β} controller. In section 4, two cases of study are presented for a position servo, to demonstrate the advantages of the tuning method. Section 5 presents robustness tests. Finally, section 6 draws the main conclusions and addresses some perspectives of future developments.

2. TOOLS AND METHODS

2.1 Control Structure

The proposed fractional order PI^{α}-PD^{β} controller is performed in two steps: the first for the output of the fractional order PI^{α} controller and the second for the output of the fractional order PD^{β} controller. By moving the fractional order PD^{β} into an inner feedback loop, an unstable or integrating process can be stabilized and then can be controlled more effectively by the fractional order PI^{α} controller in the forward path. The final fractional order PI^{α}-PD^{β} controller combines these two individual controllers together in an appropriate way. Therefore, the control structure shown in Figure .1

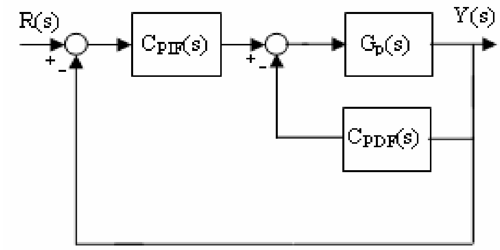

Figure 1. The fractional order PI^{α}-PD^{β} control system.

The transfer function of a fractional order PI^{α} controller is given by:

$$C_{PIF}(s) = (K_{PIF} + K_{IF} / s^{\alpha}) \quad (1)$$

Where K_{PIF} and K_{IF} are the proportional and integral gains, respectively, and $0<\alpha<1$.

The transfer function of a fractional order PD^{β} controller is given by:

$$C_{PDF}(s) = (K_{PDF} + K_{DF} s^{\beta}) \quad (2)$$

Where K_{PDF} and K_{DF} are the proportional and derivative gains, respectively, and $0< \beta<1$.

This structure, which uses an inner feedback loop, is not totally a new concept. Researchers are proposed this structure, but for the integer order PI-PD controllers [5, 13].

2.2. Rational Function of Fractional PI^{α}-PD^{β} Controller

There are many different ways to get such approximations, a good review of these approximations techniques can be found in [14]. In this work, the singularity function method [15], [16] of approximation of the fractional order operators by rational transfer function has been used. The rational function approximation of $C_{PIF}(s)$ control low, in a given frequency band of practical interest $[\omega_L, \omega_H]$, is given as:

$$C_{PIF}(s) = K_{PIF} + \left(\frac{K_I}{K_{IF}} \frac{\prod_{i=0}^{N_I - 1} \left(1 + \frac{s}{z_{Ii}}\right)}{\prod_{i=0}^{N_I} \left(1 + \frac{s}{p_{Ii}}\right)} \right) \quad (3)$$

The rational function approximation of $C_{PDF}(s)$ control low, in a given frequency band of practical interest $[\omega_L, \omega_H]$, is given as:

$$C_{PDF}(s) = K_{PDF} + \left(K_{DF} K_D \frac{\prod_{i=0}^{N_D} \left(1 + \frac{s}{z_{Di}}\right)}{\prod_{i=0}^{N_D} \left(1 + \frac{s}{p_{Di}}\right)} \right) \quad (4)$$

2.3. Hall-Sartorius Method

The Hall-Sartorius method [17] consists of finding, for a linear system, a controller minimizing the integral square error (ISE) of feedback control system for a unit step input. The integral square error (ISE) is given as:

$$J = \int_0^\infty [e(t)]^2 \, dt = \int_0^\infty [r(t) - y(t)]^2 \, dt \quad (5)$$

Where e(t)=[r(t) – y(t)] is the error signal. According to Laplace transform properties the integral J can be written as [17]:

$$J = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} E(s)E(-s)ds \quad (6)$$

Then, for E(s) a rational function in s given as $E(s) = N_E(s)/D_E(s)$, the complex integral J will be:

$$J = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} \frac{N_E(s)N_E(-s)}{D_E(s)D_E(-s)} ds \quad (7)$$

The complex integral of equation (7) is obtained as [17]:

$$J = \frac{(-1)^{(n-1)}}{2} \frac{\Delta_n^N}{\Delta_n^D} \quad (8)$$

3. DESIGN OF THE FRACTIONAL ORDER PI$^\alpha$-PD$^\beta$ CONTROLLER

3.1 Proposed Integral Criterion

In this case, for getting good dynamic performance and avoiding large control effort, we can propose a general criterion with a linear combination:

$$J = w_1 \int_0^\infty [e(t)]^2 \, dt + w_2 \int_0^\infty [u(t)]^2 \, dt \quad (9)$$

The performances criterion *J* can satisfy the designer requirement using the weighting factors w_1 and w_2 values. For the current study, selections are $w_1=w_2=1$.

3.2 Parameter Tuning of Fractional Order PI^{α}-PD^{β} Controller

The fractional order PI^{α}-PD^{β} controller has six parameters to be tuned (i.e., K_{PIF}, K_{IF}, α, K_{PDF}, K_{DF}, β).

For unit step change in the reference, we can obtain:

$$E(s)=\left(\frac{1+G_P(s)C_{PDF}(s)}{1+G_p(s)\left(C_{PIF}(s)+C_{PDF}(s)\right)}\right)\left(\frac{1}{s}\right) \quad (10)$$

$$U(s)=\left(\frac{C_{PIF}(s)}{1+G_p(s)\left(C_{PIF}(s)+C_{PDF}(s)\right)}\right)\left(\frac{1}{s}\right) \quad (11)$$

We use the Hall-Sartorius method [17] to evaluate the general integral criterion.
From equation (9), the complex integral J is given as:

$$J=w_1\frac{1}{2\pi j}\int_{-j\infty}^{+j\infty}E(s)E(-s)ds+w_2\frac{1}{2\pi j}\int_{-j\infty}^{+j\infty}U(s)U(-s)ds \quad (12)$$

Finding the appropriate setting of the values of the six parameters to achieve optimal performance for a given plant, thus calls for real parameters optimization on the six-dimensional spaces. In order to calculate the complex integral J using the Hall-Sartorius method presented in section (2.3), E(s) and U(s) must be rational functions. The approximation of the fractional PI^{α}-PD^{β} controller by a rational function is made in a given frequency band of practical interest [ω_L, ω_H].

The general criterion J is given by:

$$J=w_1\frac{(-1)^{(n-1)}}{2}\frac{\Delta_n^{N_E}}{\Delta_n^{D_E}}+w_2\frac{(-1)^{(m-1)}}{2}\frac{\Delta_m^{N_U}}{\Delta_m^{D_U}} \quad (13)$$

The settings of the six parameters of the fractional PI^{α}-PD^{β} controller consists in finding these parameters that minimize the index J of equation (9). For the minimization task, we varied the values of the six parameters each with a step of 0.01 and for each value of the six parameters; we calculate the corresponding index J. With a simple comparison test of the entire index J calculated, we can obtain the minimum index J and the corresponding optimum settings of the six parameters of the fractional PI^{α}-PD^{β} controller.

4. SIMULATION RESULTS

The following plant is used to evaluate the control performance of the proposed fractional PI^{α}-PD^{β} control structure. We consider First Order Plant plus an Integrator and Time Delay, given by [18]

$$G_p(s)=\frac{k_c}{s(\tau s+1)}e^{-Ls}=\frac{1.4}{s(0.7s+1)}e^{-0.05s}$$

The settings of the six parameters K_{PIF}, K_{IF}, α, K_{PDF}, K_{DF} and β, of the fractional PI^{α}-PD^{β} controller consists in finding these parameters that minimize the index J of equation (9), in witch we take in consideration the minimization of the energies of error and control effort.
To set the six parameters using the method proposed in section (3.2), we have first to approximate the time delay $e^{-0.05s}$ of the plant's transfer function $G_p(s)$ by a rational function using the Padé approximation method. Then, we approximate the above fractional PI^{α}-PD^{β} controller by a rational function in the

frequency band $[\omega_L,\omega_H] = [\omega_L=0.1\omega_u, \omega_H=10\omega_u]$ = [.23 rad/s , 23 rad/s] .
The smallest index J is obtained for the parameters K_{PIF}= 0.432, K_{IF}= 2.005, K_{PDF}= 0.701, K_{DF}=2.6025, α=0.64, β=0.74. Hence, the fractional order PI^{α}-PD^{β} controller's output signal $U_2(s)$ is given as:

$$U(s) = \left(0.432 + \frac{2.005}{s^{0.64}}\right)E(s) - \left(0.701 + 2.6025 s^{0.74}\right)Y(s)$$

In this case, the fractional $PI^{0.64}$-$PD^{0.74}$ controller tuned by minimization of the index J is compared with that tuned by using Ziegler-Nichols second method given by:

$$C_{ZN}(s) = 9 + \frac{10.2623}{s} + 1.9732 s$$

Also, the fractional $PI^{0.80}$-$PD^{0.75}$ controller is compared with the integer PI-PD controller tuned by minimization of the index J given by:

$$U(s) = \left(1.00 + \frac{3.05}{s}\right)E(s) - \left(9.21 + 2.32 s\right)Y(s)$$

Figures (2) showS the step responses the closed loop systems. As can be seen from Figure (2) the proposed fractional $PI^{0.64}$-$PD^{0.74}$ controller outperforms the other three controllers, and an acceptable step response is traced.
Some performances characteristics for the feedback control systems with the fractional $PI^{0.64}$-$PD^{0.74}$ controller, C_{ZN} and PI-PD are summarized in Table.1, in terms of unity gain crossover frequency w_u, phase margin PM, gain margin GM, rise time T_r. settling time T_s, overshoot P, index J_1 and J.
As can be seen from Table.1, the proposed fractional $PI^{0.64}$-$PD^{0.74}$ controller gives the low index J, this because the controller is tuned for this purpose.

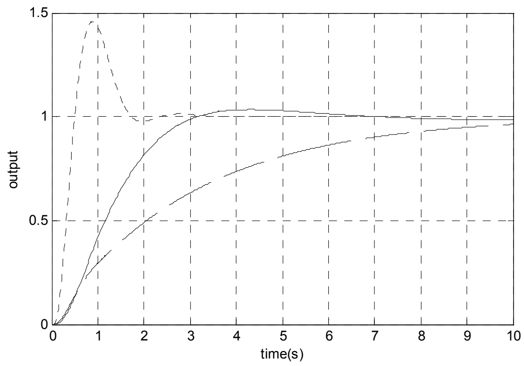

Figure 2. Step responses of the closed loop system with $PI^{0.64}$-$PD^{0.74}$ (solid line), C_{ZN} (dotted line) and PI-PD (dashed line).

Table 1. Temporal characteristics.

Controller	T_r (sec)	T_s (sec)	P (%)	J_1	J
C_{ZN}	0.2	2.0	38	0.3624	18.11
PI-PD	6.55	11.8	0	0.7717	01.32
$PI^{0.64}$-$PD^{0.74}$	1.9	5.6	3.0	0.2572	01.29

5. ROBUSTNESS TESTS

From figure 3 of the study, it can be concluded that the compensated systems using the proposed fractional $PI^{0.64}$-$PD^{0.74}$ controllers are robust to changes of the static gain. In short, it can be said that the use of the fractional $PI^{0.64}$-$PD^{0.74}$ controllers provide better responses and robust systems.
In order to test to test the fractional $PI^{0.64}$-$PD^{0.74}$ controller in the presence of noise, the above process is tested for two different variances σ^2=0.0001. s it can be seen from figure 4 the fractional $PI^{0.64}$-$PD^{0.74}$ controller shows very good noise rejection feature.

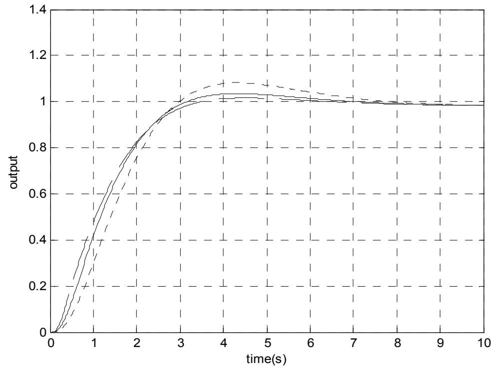

Figure.3. Step responses of the closed loop system with a fractional $PI^{0.64}$-$PD^{0.74}$ controller with static gain variations: K_c=0.7 (dotted line), K_c=1.4 (solid line) K_c=2.8 (dashed line)

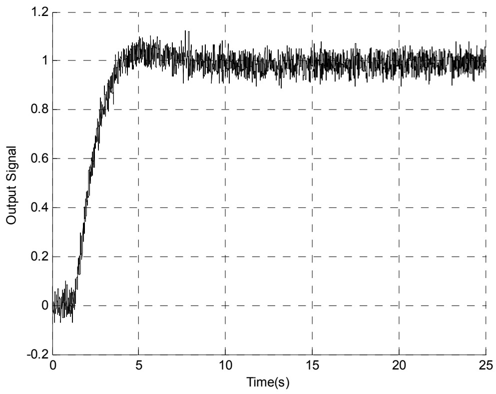

Figure.4. Step response of the closed loop system for σ1=0.00001

6. CONCLUSION

In this study, fractional order PI^{α}-PD^{β} controller has been introduced. The performance of this tuning method for the fractional PI^{α}-PD^{β} controller tuning was tested in different situations (varying the steady state gain). The fractional PI^{α}-PD^{β} controller showed high robustness in all cases compared to the conventional controllers. The fractional PI^{α}-PD^{β} controller is applied to a model with noise. Even in the presence of very high noise, the fractional PI^{α}-PD^{β} controller showed good control behaviour.

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