Development of a simulation model for a three-dimensional wind velocity field using Ténès Algeria as a case study

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A B S T R A C T
A mathematical simulation model was developed that can determine the three-dimensional wind velocity field over a complex terrain. The Ténès area in the Valley of Chelif in Algeria was used as a case study. This region is exposed to south-west circulation that makes it favorable to the use of wind energy. Knowledge of wind fields is crucial for predicting the dispersion of pollutants, for forecasting meteorological weather, for fire spread prediction and in the design and implementation of wind turbines. By means of a mass consistent model, an in-house program was developed to calculate the three-dimensional wind velocity field in the study region. The model was supported by a numerical box in which flow through is allowed for in the upper and lateral boundaries. The bottom boundary through which no flow through occurs was determined by the topographic relief at the surface. From measured wind velocities, observed values were calculated by interpolation–extrapolation. Using an optimization method, the adjusted velocities were obtained from constraints, observed velocities and the continuity equation. The model was verified with wind point data, the relative error did not exceed 6%.

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1. Introduction

The interest in wind energy production has increased considerably due to improvements in turbine technology and the growing need for power production from renewable sources [1–3]. One of the most important issues to deal with is the choice of an adequate site for a wind farm [4]. Different methodologies have been developed to investigate and define the wind climate of regions using
actual measured data [5]. Mortensen and Petersen [6] for example have shown that numerical models are suitable for simulating wind fields. Such models can indicate the best sites for installation of wind turbines.

Due to a boost in the world’s energy consumption as a result of population growth and industrialization, renewable energy (i.e. solar, wind, geothermal and wave) has increased in importance [7]. A study by Ludwig and Byrd [8] has shown that meteorological and air pollution problems frequently require the interpolation of complete, three-dimensional wind field flows from a limited number of observations at specific location. Modeling atmospheric flow is difficult, especially when simulating wind fields over irregular terrain [9].

Different types of wind field models have been developed in the last few years [10]. They can be classified into two main models: diagnostic (kinetic) and prognostic (dynamic). Kinetic models are those capable of reconstructing a steady-state wind field starting from a set of initial experimental data. Their equations contain no time dependent terms. On the other hand, dynamic models describe the time evolution of meteorologically variable fields starting from an initial state and merging the effects of possible time variations of boundary conditions.

Diagnostic (i.e. kinetic) models follow two main approaches [11]: a linearized model which is a simplified steady-state solution of the Navier–Stokes equation, and the mass-consistent model which analyses available meteorological data, with physical constraints like mass-conservation. It is a general practice to use mass consistent models for simulation of stationary three-dimensional wind fields in complex terrain. The advantage of a mass consistent model compared with primitive equation models is the relatively short computing time. However, as a result it is not possible to look at complex phenomenon such as turbulence [12].

The implementation of a mass consistent wind field is typically undertaken using available meteorological data. The procedure has been effective for predicting winds for emergency response contaminant dispersion, for weather forecasts, in wind energy studies and for storm flooding prediction [13]. Furthermore, some applications require that the derived wind field be non divergent.

The mass-consistent models reconstruct 3-D wind fields starting from vertical profiles and near-ground wind measurements through a two-step procedure [14]. The observed data needed for the mass consistent model are provided by an interpolation–extrapolation scheme using information available at a given site to determine velocity components at each grid point above the topography. Typically the available simultaneous data consists of several horizontal wind speed and direction measurements near the earth’s surface, a vertical profile of horizontal wind speed and direction and synoptic analysis.

The wind data interpolation method plays a crucial role in determining the final wind field features. Different wind fields can be generated starting from the same data and changing only the interpolation method. The adjustment procedure is generally obtained by employing the variational analysis technique [15]. Early research was conducted by Sasaki [16–18] who developed the first theoretical model for the calculation of wind speed. Many different mass-consistent models have been developed starting from the late seventies [19–29]. Various other works relating the results of air pollution dispersion modeling in complex terrain are entirely dependent on adequate descriptions of the three-dimensional wind field [30]. Furthermore, as part of modernization efforts of the Atmospheric Release Advisory Capability (ARAC) project, Chan and Sugiyama [31] developed a new diagnostic model for generating mass consistent wind fields over a continuous terrain. In addition, Leone et al. [32] have used wind fields to drive ARAC’s new dispersion model. This model is going to replace the current operational code described by Sherman [19] which employs stair-step topography and constant grid spacing. The topography strongly influences the boundary layer flow in such a manner that the geographic variations either can modify on in some cases generate circulations in conjunction with diurnal heating cycles [33, 34].

The theoretical basis for the mass consistent model was developed by Sasaki [16–18]. The general variational analysis formalism defines an integral function whose external solution minimizes the variance of the difference between the observed and analyzed variable values subject to physical constraints which are the continuity equation and the measured velocities.

The aim of this paper is to develop a mathematical simulation model that can determine the three-dimensional wind velocity field over a complex terrain using the Valley of Chelfif in the Tenes area in Algeria as a case study. This region has a complex relief with a set of mountains and valleys with strong slopes. In the adjustment model for wind flow, the relief is analyzed explicitly with the use of velocity measurements in the field using a large number of points in a three-dimensional grid. The data are interpolated and extrapolated by a least squares method on the grid of an Euler model. The numerical results are compared with the measurement values at selected points on the real terrain.

### 2. Methodology

#### 2.1. Case study area

The study area is located in the town of Tenes in Algeria. It is a coastal city which is partially surrounded by the Dahra mountain range. The topography of this region is complex with a variation in altitude between 0 and 700 m (Fig. 1a). The correspondent mathematical topographic model is given in Fig. 1b. The data file that defines the terrain relief by the three coordinates contains 374,037 points. Digital processing was used to order the relief structure. The site is characterized by an important wind potential area [35]. The weather station at this location has 5 years of continuous measurements of three-hourly speed and wind direction [36]. Characteristics of the measuring station are given in Table 1.

#### 2.2. Mathematical model formulation

The study is presented as an optimization problem with constraint conditions. The objective function is given by an integral
The functional $[16-18]$:  

$$E = \int_V (F(u, v, w, \lambda) \, dx \, dy \, dz \tag{1}$$

where $F$ is a function expressed by

$$F(u, v, w, \lambda) = \alpha_u^2 (u - \bar{u})^2 + \alpha_v^2 (v - \bar{v})^2 + \alpha_w^2 (w - \bar{w})^2 + \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \tag{2}$$

where $u$, $v$, and $w$ are the argument functions which represent respectively the components wind velocity in $x$, $y$, and $z$ direction of the modeling domain $V$. The mass consistent model constructs an observed initial guess of the wind velocities component $u^0$, $v^0$, and $w^0$ over the entire model. The initial estimates are formed from an interpolation–extrapolation scheme that uses existing surface wind observations contained within the modeled area. The values $u$, $v$, and $w$ are obtained by adjusted scheme from the observed velocities by minimizing the functional $E$. The parameter $\lambda(x,y,z)$ is the Lagrange multiplier; and values of $\alpha_i$ are Gauss precision moduli taken to be $\alpha_i^2 = 1/2\sigma_i^2$. $\sigma_i$ is a measurement error and/or variances of the observed field from the desired adjusted field. Identical Gauss precision moduli are assumed for the horizontal directions. The ratio $\alpha = (\alpha_1/\alpha_2)^2 = (\sigma_2/\sigma_1)^2$ is related to stability and turbulence of atmosphere [19].

The variational calculus gives the Euler–Lagrange equation for $u$, $v$ and $w$, respectively [37]:

$$\frac{\partial F}{\partial u} - \frac{d}{dx} \left( \frac{\partial F}{\partial u_x}\right) = 0 \tag{3}$$

$$\frac{\partial F}{\partial v} - \frac{d}{dy} \left( \frac{\partial F}{\partial v_y}\right) = 0 \tag{3}$$

$$\frac{\partial F}{\partial w} - \frac{d}{dz} \left( \frac{\partial F}{\partial w_z}\right) = 0 \tag{3}$$

where

$$u_x = \frac{\partial u}{\partial x} \tag{4}$$

$$u_y = \frac{\partial u}{\partial y}$$

$$u_z = \frac{\partial u}{\partial z}$$

Eq. (3) are subjected to the boundary conditions:

$$\pi \lambda \nabla \bar{u} = 0 \tag{5}$$

where $\bar{u}$ is the outward positive unit normal to the boundary of volume $V$. Eq. (5) implies that on the boundary, either of the following is satisfied:

$$\nabla \bar{u} = 0 \quad \text{or} \quad \lambda = 0 \tag{6}$$

Eq. (5) represents appropriate conditions for “flow-through” lateral and upper boundaries. Either the normal velocity component variation or $\lambda$ must be zero on a boundary. The appropriate condition for “flow-through” boundary (upper and lateral boundary) is $\lambda = 0$. In this case the derivative of $\lambda$ in the direction across the boundary is in general not zero. The lower boundary coincides with the ground and represents a “no-flow-through” solid boundary. The observed velocity at that boundary is zero $[19,25,42]$.

Substituting for $F$ by putting Eq. (2) into Eq. (3), we obtain:

$$u = u^0 + \frac{1}{2\alpha_1^2} \frac{\partial \lambda}{\partial x} \tag{7}$$

$$v = v^0 + \frac{1}{2\alpha_1^2} \frac{\partial \lambda}{\partial y} \tag{7}$$

$$w = w^0 + \frac{1}{2\alpha_2^2} \frac{\partial \lambda}{\partial z} \tag{7}$$

The continuity equation is given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{8}$$

The equation for $\lambda$ is derived by differentiating Eq. (7) and substituting the results into Eq. (8) giving:

$$\frac{\partial^2 \lambda}{\partial x^2} + \frac{\partial^2 \lambda}{\partial y^2} + \frac{\partial^2 \lambda}{\partial z^2} = -2\alpha_1^2 \frac{\partial u^0}{\partial x} + \frac{\partial v^0}{\partial y} + \frac{\partial w^0}{\partial z} \tag{9}$$

Eq. (9) is an elliptical differential equation (Poisson’s equation). It will be solved for $\lambda$ with the specific boundary conditions of Eq. (5). The adjusted wind velocity components are calculated using Eq. (7).

2.3. Method of resolution

This case study can be described as a constraint optimized problem where Eq. (1) is the objective function and the constraint conditions are the continuity equation and the measured values in the volume $V$. Mass-consistent optimization is used to solve the problem. The basic procedure to get the wind field velocities by the mass-consistent method is given by the following steps:

The Lagrange parameter $\lambda_{ijk}$ is determined by Eq. (9) by the finite difference method in the discretised volume $V$ which is a rectangular box set on the topographic relief. The grid model is built with a choice of $\Delta x$ and $\Delta y$ such that the grid takes in account
the more interesting topographic detail features of the site (e.g., mountains, canyons and valley areas). The computational volume is shown in Fig. 1.

By using a central difference scheme for second derivatives for a grid point $(i,j,k)$, Eq. (9) is rewritten as:

$$
\frac{\lambda_{i,j+1,k} - 2\lambda_{i,j,k} + \lambda_{i,j-1,k}}{\Delta x^2} + \frac{\lambda_{i+1,j,k} + \lambda_{i-1,j,k} - 2\lambda_{i,j,k}}{\Delta y^2} + \frac{\lambda_{i,j,k+1} + \lambda_{i,j,k-1} - 2\lambda_{i,j,k}}{\Delta z^2} - 2\alpha_l^2 \varepsilon_0 = 0
$$

(10)

With the divergence $\varepsilon_0$ of the interpolated wind field is given by:

$$
\varepsilon_0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}
$$

(11a)

Using central difference for the first derivates $\varepsilon_0$ is written as:

$$
\varepsilon_0 = \frac{u_{i+1,j,k} - u_{i-1,j,k}}{2\Delta x} + \frac{v_{i,j+1,k} - v_{i,j-1,k}}{2\Delta y} + \frac{w_{i,j,k+1} - w_{i,j,k-1}}{2\Delta z}
$$

(11b)

where the observed values $u^0_{i,j,k}$, $v^0_{i,j,k}$ and $w^0_{i,j,k}$ were determined by least square weighted method from the measured wind speed. The method uses the three nearest measured points to determine the observed wind speed at each grid point $(i,j,k)$. Once the observed velocities are determined, $\varepsilon_0$ is introduced in Eq. (10). A set of difference equation in $\lambda_{i,j,k}$ is now obtained at each interior grid point. The entire system is solved simultaneously for the value of $\lambda_{i,j,k}$. The equation set is solved iteratively, using a successive over-relaxation method.

In altitude $z$ the observed values (either measured or adjusted values) are extrapolated with the following equation [38]:

$$
\bar{U} = \bar{U}_0 \left( \frac{z}{z_0} \right)^p
$$

(12)

where the exponent $p$ is determined by atmospheric stability conditions or from a least squares fit to multiple level tower data.

Once the Lagrange parameters are determined, the adjusted wind field values are calculated using central difference for $\lambda_{i,j,k}$ and mean value for observed wind velocity of Eq. (7):

$$
\begin{align*}
\lambda_{i,j,k} & = \frac{1}{4} \left( \frac{u_{i+1,j,k} + 2u_{i,j,k} + u_{i-1,j,k}}{2\Delta x} + \frac{v_{i,j+1,k} + 2v_{i,j,k} + v_{i,j-1,k}}{2\Delta y} + \frac{w_{i,j,k+1} + 2w_{i,j,k} + w_{i,j,k-1}}{2\Delta z} \right) \text{ (13)}
\end{align*}
$$

The algorithm for calculating the adjusted wind speed values is determined as follows:

- Obtain three-dimensional topographic data of the complex terrain.
- Develop three-dimensional discrete reliefs for finite difference method.
- Determine 3-D numerical box for finite difference method application.
- Localize measured values of wind velocities on the discrete bottom boundary of the box.
- Determine the observed wind velocities ($\bar{u}^0$, $\bar{v}^0$ and $\bar{w}^0$) by least-squares method.
- Calculate the Lagrange parameters in the volume box by finite difference method.
- Use SOR method to resolve set of equations compute adjusted velocity components ($u$, $v$ and $w$) within the box by Euler–Lagrange method (solution of the problem by variational form).

### 3. Results and discussion

#### 3.1. Numerical development of the terrain and grid generation

By letting $x$, $y$ represent the horizontal direction and $z$ the vertical direction over a modeling domain $V$, the data are arranged in the $y$ direction and subdivided in columns. The choice of the grid size is often determined by the necessity to show the real topographic features. Fig. 2 shows a cross section of the computational volume. The topography may protrude through the top of the grid potentially producing unconnected regions within the grid volume. This feature can be used to study wind fields in valleys, canyons and mountain areas. The numerical relief shown in Fig. 3 indicates that the area of interest for this model is a rectangular box set on the earth’s surface with the bottom of the box located at the lowest topographic point. The dimensions of the box are determined by the application requirements and computer storage limitations.

The computational domain is meshed by $18 \times 6 \times 10$ nodes with step $\Delta x = \Delta y = 2000$ m and $\Delta z = 100$ m, as shown in Fig. 3. The values of $\sigma_1$ and $\sigma_2$ were taken to be 1 and 0.01 ms$^{-1}$, respectively. The wind speed and direction at the upper boundary of the grid were temporally interpolated from the two vertical profile observations and assumed to be spatially constant. The horizontal wind measurements were adjusted to a height of 50 m above the terrain using a power law exponent of 0.2 [39]. The constant height interpolated and extrapolated wind field is processed to define horizontal wind components at each grid point about the topography boundary. The foregoing applications differ in terrain, meteorological data availability, meteorological condition, grid resolution, and convergence criterion.

#### 3.2. Observed wind speed and Lagrange parameters

The measured values are extracted from results of three-dimensional observed velocities ($u^0_{i,j,k}$, $v^0_{i,j,k}$, $w^0_{i,j,k}$) calculated by the AILOS software [40]. Fig. 4 shows 11 measured points (black symbols) which were chosen on the site at 10 m from the surface. The discretised Eq. (9) is solved to give the Lagrange parameters values $\lambda_{i,j,k}$ at each node of the 3D-grid. The matrix coefficients of the equation system are diagonally dominant and asymmetric. The iterative successive over relaxation, SOR, method [37] was used to solve the correspondent equation system. Eq. (9) can be written as:

$$
\begin{align*}
\lambda_{i,j,k}^{n+1} & = (1 - \omega)\lambda_{i,j,k}^n + \omega \frac{1}{A_{i,j,k}} \left[ A_{i,j,k} + \lambda_{i,j,k}^{n-1} + \lambda_{i-1,j,k}^{n-1} + \lambda_{i+1,j,k}^{n-1} + \lambda_{i,j-1,k}^{n-1} + \lambda_{i,j+1,k}^{n-1} + \lambda_{i-1,j-1,k}^{n-1} + \lambda_{i-1,j+1,k}^{n-1} + \lambda_{i+1,j-1,k}^{n-1} + \lambda_{i+1,j+1,k}^{n-1} \right] \\
\end{align*}
$$

(14)

where

$$
A_{i,j,k} = 2(\alpha_1 \Delta x)^2 \varepsilon_0
$$

(15)
difference of over 500 m between the highest and the lowest layer was observed. The model complex relief is composed of a set of mountains or hills with a high slope. The major characteristics of the flow over a hill were determined not only by the hill shape but also by its size and by atmospheric stability. The horizontal wind measurements shown by the colored contour lines in Fig. 4 were taken from the observed velocities calculated by AILOS software in the study region of Tenes. The accuracy of the wind field is dependent on the specification of $\alpha_1$ and $\alpha_2$. For a stable atmosphere the horizontal adjustment were given by $\alpha_1 = 1$ and $\alpha_2 = 10$; then the adjustment is predominant in the horizontal component instead of vertical component since:

$$\alpha = \left(\frac{\alpha_1}{\alpha_2}\right)^2 = k \cdot \left(\frac{\bar{w}}{\bar{u}}\right)^2 = 10^{-2}$$

(19)

The developed computer code, applied on the numerical relief in shown in Fig. 3, produces nondivergent three-dimensional wind at 1080 grid points with the use of a Pentium 4, CPU 2.66 GHz.

3.3.1. Wind field velocity in horizontal planes

Fig. 5a shows the horizontal wind field velocity at plane $(x, y)$ above the topographic relief. The wind values were adjusted with a step height of 100 m in the $z$ direction using Eq. (12). The exponent $p$ has a value of 0.2 in a period where the atmosphere was neutrally steady. The highest wind velocities were observed in the intervals [364,366] [352,356] in the $x$ direction. An interesting area is a valley with a higher depth from the reference sea level. Fig. 5b gives the correspondent wind field which is represented by isolines of constant velocity. This gives a more precise view of the wind field velocity in the horizontal plane $(x, y)$ than the representation of velocity given by Fig. 5a. A maximum value of the velocity reached 6 m/s in the valley at $x = 366$ km. It was observed that wind direction reversal occurs when the wind shifts across the valley. This outcome is also predicted by Finardi et al. [13] for a relief with a single valley. The results obtained in our study for different horizontal planes show that the rates decrease gradually as the altitude increases and become regular at an altitude of 1000 m. This is consistent with the results obtained by Sherman [19].

3.3.2. Wind field velocity in vertical planes

Fig. 6a–e show the velocity distribution in a vertical plane section of the computational volume and in particular the vertical motion resulting from terrain features and the wind shear present

$$B = \alpha \left(\frac{\Delta x}{\Delta z}\right)^2$$

(16)

$$C = 4 \left[1 + \alpha \left(\frac{\Delta x}{\Delta z}\right)^2\right]$$

(17)

where $\Delta y = \Delta x$, the coefficients $A_{ijk}$, $B$ and $C$ have known values. The variable specifies the current value of the iteration in the SOR method.

The relaxation factor $\omega$ was determined experimentally and was equal to 1.78. This value is chosen so as to obtain the minimum number of iterations. The iterative solution is considered converged when the relative change of every $\lambda_{ijk}$ is less than a prescribed value. The iterative process is stopped when the condition on the solution accuracy is achieved

$$\left|\lambda_{ik}^{it} - \lambda_{ik}^{it-1}\right| < \varepsilon$$

(18)

where $\varepsilon$ is a given accuracy value.

3.3. Wind speed adjusted values

The numerical technique was applied on the model as shown in Fig. 3. The study terrain was very rough, an elevation

Fig. 3. Three-dimensional grid computational volume.

Fig. 4. Observed velocities (colored lines) calculated by AILOS software [40] and the measured points (11 black symbols). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)
Fig. 5. Adjusted horizontal wind field in the (x, y) plan above the topography.

in the vertical profile. The field velocity decreased when the altitude increase as predicted by Equation (12); the highest velocities are located at the wall (flank) of the mountain and the area of the valley located in the intervals [364,366] [352,356] as described in section of horizontal velocities. This area indicates the location of highest power wind. These results are similar to those predicted by the two main diagnostic models; MATHEW, developed by Sherman [19] and MINERVE, developed by Geai [24] which appear to have been the more accurate and the most extensively used.

Table 2 shows the comparison of wind speeds (m/s) estimated using the current model and that estimated using Wasp for the measuring station area Tenes [41]. The current model gave a value
Table 2
Comparison of wind speeds estimated using the current model with that estimated by Wasp [41].

<table>
<thead>
<tr>
<th>Coordinate of measurement point (m)</th>
<th>Present study (m/s)</th>
<th>Wasp (m/s) [41]</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X = 3774356, Y = 356412.065, 0)</td>
<td>3.47</td>
<td>3.69</td>
<td>−6%</td>
</tr>
</tbody>
</table>

of 3.47 m/s whereas the Wasp method gave a slightly higher value of 3.69 m/s [41]. It is noted that the result of the estimate based on the current model is lower by 6% compared with that predicted by Wasp.

4. Concluding remark

A numerical algorithm for the calculation of flow fields was developed. The study showed that basic optimization of constraint wind flows using a mass-consistent model is an effective method for obtaining a 3-D wind simulation over complex terrain. An application was successfully adopted for a complex terrain in the region of Tenes in Algeria. The current model gave wind speed values that were 6% lower than those estimated by Wasp software. The method allowed us to determine the windiest areas in the case study region. The precision in modeling complex terrain and the application of the scheme difference method on the topographic boundary...
suggest a significant influence on the wind speed. The model developed can also be applied to dispersion of pollutants, and prediction of fire spreading.

References


