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A numerical analysis of relationship between ductility and nucleation and critical void volume fraction parameters of Gurson–Tvergaard–Needleman model

M Hadj Miloud¹,², A Imad³, N Benseddiq³, B Bachi Bouiadjra⁴, A Bounif² and B Serier⁴

Abstract
Gurson–Tvergaard–Needleman model is widely used to describe the three stages of ductile tearing: nucleation, growth and the coalescence of micro-voids. The aim of this article is to study the relationship between volume fraction of voids and the fracture strain $e_f$. The effects of the volume fraction of nucleation, $f_N$, and the critical volume fraction, $f_c$, were analysed. These parameters play crucial roles in the process of ductile damage. A phenomenological analysis is carried out to study the relationship between the different void volume parameters and the fracture strain $e_f$. A method is proposed for the determination of $f_N$ and $f_c$, knowing the experimental fracture strain $e_f$. The experimental parameters are extracted from the load–diametric contraction curve of an axisymmetric notched tensile bar test AN2.

Keywords
Damage mechanics, Gurson model, parameters identification, parametric analysis

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Introduction
Metallographic studies¹,² demonstrate that ductile damage of a metal is basically characterised by three mechanisms of void growth: (a) nucleation of voids due to fracture of particle–matrix interface, failure of the particle or micro-cracking of the matrix surrounding the inclusion; (b) growth of voids leading to an enlargement of existing holes; and (c) coalescence or micro-cracks initiated from voids leading to the drop of the load-carrying capacity of the material, when the void volume fraction (VVF) approaches unity.

One of the best known micro-mechanical models describing the damage of metallic ductile materials is that of Gurson.³ Based on the works of Rice and Tracey⁴ and McClintok,⁵ Gurson further studied the plastic flow around voids in a metallic material.

The Gurson model was derived by assuming a homogeneous deformation field in the matrix surrounding the void. He developed a constitutive model for porous ductile media based on a rigid-plastic material behaviour and the upper bound theorem of plasticity. This model is based on detailed phenomenological studies of the bifurcation behaviour in materials with periodic distributions of cylindrical and spherical voids

$$\Phi(\Sigma, \sigma, f) = \frac{\sigma_{eq}}{\sigma_0^2} + 2\cosh\left(\frac{3}{2} \frac{\sigma_{eq}}{\sigma_0}\right) - (1 + f^2) = 0$$

where $\sigma_{eq}$ is the macroscopic equivalent von Mises stress and $f$ the VVF considered as a damage parameter.

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Tvergaard\textsuperscript{6,7} refined the constitutive model by incorporating some additional parameters \((q_1, q_2)\), resulting in the following yield function

\[ \Phi(\Sigma, \bar{\sigma}, f) = \frac{\sigma_m^2}{\sigma_f^2} + 2q_1f^* \cosh\left(\frac{3}{2}q_2 \frac{\sigma_m}{\sigma_f}\right) - \left(1 + q_2^2 f^*_2\right) = 0 \]  \hspace{1cm} (1)

Note that Tvergaard has originally suggested the values of the additional parameters as \(q_1 = 1.5\) and \(q_2 = 1\). Perrin and Leblond\textsuperscript{8} have determined a correlation between the parameter \(q\) and porosity fraction \(f\); \(q = q(f)\). These authors showed that when the porosity fraction tends to zero, the value of \(q\) tends towards 1.47. Faleskog et al.\textsuperscript{9} indicated that the two parameters \((q_1, q_2)\), depend on the material hardening exponent. Steglich and Brocks\textsuperscript{10} have proposed a yield condition for nodular cast iron. The condition was determined from a cell model which results from using a quadratic equation to estimate the parameter \(q_1\). Moreover, Kim et al. and Ragab\textsuperscript{11} showed that the \(q\) parameters should be varied with the triaxiality of the stress field, as well as the initial porosity. Vadillo and Fernández-Sáez\textsuperscript{12} used a consistent fully implicit field, as well as the initial porosity. VVF evolution.

Figure 1. VVF evolution. VVF: void volume fraction.

where \(\delta\), can be expressed as \(\delta = \frac{f^*_0 - f^*}{f^*_0}\), denotes the coalescence accelerator factor, where \(f^*\) is the current volume fraction of the voids; \(f^*_0\) and \(f^*_c\) denote, respectively, the VVFs at failure and the critical situation in which \(f^*\) starts to deviate from \(f^*_c\), and \(f^*_0\) is the value of \(f^*\) at fracture condition (i.e. 2). (see figure) The load-carrying capacity vanishes when \(f^* = 1/\{f^*_0 - f_c\} = 1/q_1\).

Generally, the initial VVF, \(f^*_0\), is evaluated by microscopical examination of the undamaged material; it is a parameter that characterises the initial state of the material. The critical VVF, \(f_c\), signifies the onset of coalescence. In most investigations, only the critical VVF, \(f_c\), is considered as a material parameter, obtained by fitting the numerical calculations with the experimental results, and the other parameters are fixed arbitrary. According to Zhang and Niemi,\textsuperscript{14} \(f_c\) is not constant but decreases when the stress triaxiality ratio \(T\) increases. However, other authors note that \(f_c\) can be taken as a constant only for small values of \(f_0\). In a similar study, Steglich and Brocks\textsuperscript{15} confirmed that the value of \(f_c\) depends on stress triaxiality \(T\): \(f_c\) decreases with the increase of \(T\).

They are several methods to determine the critical VVF, \(f_c\). One is using cell models developed by Koplik and Needlemann,\textsuperscript{16} Kuna and Sun\textsuperscript{17} and Brocks et al.\textsuperscript{18} This method is limited because it can be used only for a given initial VVF and it does not take into account the nucleation phase. The intermediate void nucleation process is very important in ductile fracture, and it is very difficult to incorporate into cell models. Sun et al.\textsuperscript{19–21} have suggested that \(f_c\) can be numerically obtained from smooth axisymmetric tensile tests and then applied to a more general stress status case with the advantage of including the void nucleation.

Many authors have used the classical best fit methods to calibrate \(f_c\) by combining notched tensile bar tests and finite element (FE) analyses.\textsuperscript{22–25} Zhang et al.\textsuperscript{26} and Acharyya\textsuperscript{27} determined \(f_c\) using a physical micro-void coalescence criterion based on...
Thomason.2 Rakin28–30 has determined the parameter of the plastic limit load model initially developed by Thomason.28 Rakin28–30 has determined the parameter $f_c$ from an axisymmetric smooth specimen test. The critical values were determined based on the bisection of the formed curves and the straight line corresponding to the value of the diameter reduction at failure. This simple procedure seems to be closer to practical application than ‘adjustment’ (fitting) of numerical curves with the experimental ones for $f_c$–$\delta_d$ diagram (Figure 2). On the other hand, the final failure VVF, $f_f$, is considered a parameter that may be experimentally determined.31 Even though the VVF at final fracture, $f_F$, has been considered as an unimportant parameter, it is interesting to know whether it is constant.32 The increases in porosity may in general have contributions from two processes: the growth of existing voids and the nucleation of new voids. Mathematically this can be expressed as

$$\dot{f} = \dot{f}_{growth} + \dot{f}_{nucleation}$$

Assuming that the matrix is plastically incompressible; the growth part is directly related to the mesoscopic plastic dilatation

$$\dot{f}_{growth} = (1-f)\dot{e}_{kk}^p$$

where $\dot{e}_{kk}^p$ is the trace of the macroscopic plastic strain rate tensor. Void nucleation based on plastic straining can be included by

$$\dot{f}_{nucleation} = A\dot{\varepsilon}^p$$

The parameter ‘$A$’ follows a normal distribution, as suggested in Chu and Needleman32 and Tvergaard and Needleman33

$$A = \frac{f_N}{S_N \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left( \frac{\dot{\varepsilon}^p - \varepsilon_N}{S_N} \right)^2 \right]$$

Growth of porosity due to the nucleation voids introduces the parameter ‘$A$’ in equation (7); $\varepsilon_N$ is the mean void nucleation strain, $S_N$ the standard deviation of the distribution and $f_N$, the volume fraction of particles available for void nucleation. The description of this stage needs the determination of these three parameters. Generally, $\varepsilon_N$ and $S_N$ values are, respectively, 0.3 and 0.1 for most materials. Thus, they can be considered as constants of the statistical distribution law and not as intrinsic material characteristics. The parameter $f_N$, which is commonly interpreted as the volume fraction of ‘void nucleating particles’ may only be identified with the total volume fraction of the inclusions for debonding particles where the initial void contains the whole particle.34 The difficulty for modelling this problem is in introducing the nucleation of voids at cracked particles. An empirical approach was introduced in Chu and Needleman32 to describe a normally distributed nucleation density function of the three parameters, $f_N$, $\varepsilon_N$ and $S_N$.

Void growth may be treated independent of the material (hardening); however, void nucleation is a highly material-dependent process. This justifies the fact that void nucleation is the least understood part of ductile fracture. Void nucleation belongs to material intrinsic properties and governs the material failure behaviour. In general, it depends on particle strength, size and shape and the hardening exponent of the matrix material. The nucleation mechanism can be controlled by strain, or by stress; in this last case, the hydrostatic tension stress plays an important role.34

A great deal of experimental work has been performed to quantify void nucleation and growth. In general, these studies involved the use of both smooth and notched tensile specimens.32–39 The quantification of the VVF is made by both optical methods40,41 and Archimedes' principle.42–46 Modelling of the first phase of ductile fracture – void nucleation – has been carried out using quantitative metallographic analysis of non-metallic inclusion content in tested steel.39 Lievers et al.37 proposed a novel method for the quantification of void nucleation rates in sheet material. An incremental sheet forming process is used to create large regions of homogeneous deformation, such that material density variations can then be used to quantify the evolution of VVF with the applied strain.

One of the most popular and convenient ways to model the nucleation, growth and coalescence of voids in a continuum FE method formulation is the use of Gurson–Tvergaard–Needleman (GTN) constitutive softening equations. In the GTN model, nucleation is most commonly seeded using the normal distribution scheme originally proposed by Chu and Needleman.32 Huber et al.38 developed a micromechanical model for the ductility of plastically deforming materials containing a homogeneous
distribution of brittle inclusions for aluminium alloys. The model includes a micro–macro void nucleation condition with initial penny shape nature of the voids and also the growth and coalescence regimes. The model was validated by comparing the numerical predictions to the experimental results obtained under different levels of stress triaxiality and for different heat treatments controlling the hardness of the matrix.

In this article, the GTN template parameter identification methodology is proposed to describe the ductile fracture of metals. Adjustment settings of \( q_1 \) and \( q_2 \) are set to 1 and 1.5. Similarly, \( e_N \) and \( S_N \) nucleation settings are set to 0.3 and 0.1. The initial volume fraction is considered a metallurgical characteristic of the initial state of the material. It is obtained by measuring and in our case the value is equal to \( 10^{-5} \) for 12NiCr6 steel. This value is located in the low values of the \( f_0 \) range. Assumptions consider the \( f_0 \) is a parameter that characterises the end of the process of failure and its value can be approximately in the range 10–20%. Thus, our study is focused on the effect of two principal parameters, \( f_N \) and \( f_c \), on the behaviour of the fracture strain by a tensile test performed on an axisymmetric notched bar AN2.

The initial VVF is obtained from experimental observations and its value is \( f_0 = 1 \times 10^{-1} \). The final VVF is as a result of a calibration procedure. The other parameters are set as follows: \( q_1 = 1.5 \), \( q_2 = 1 \), \( e_N = 0.3 \) and \( S_N = 0.1 \).

### Materials and experimental procedure

#### Materials

The nickel and chromium steel used in this study is the 12NiCr6 steel. Its chemical composition is given in Table 1. The steel was austenitised at 880°C for 1 or 1 h, and then quenched in an oven. The initial VVF, \( f_0 \), is small (\( f_0 = 10^{-3} \)). This micro-structural parameter is necessary to use the GTN model. The principal mechanical characteristics are carried out from three tensile tests of smooth round bar with 11 mm diameter (Figure 3). These properties are summarised in Table 2.

#### Tensile procedure

Tensile tests have been conducted on axisymmetric notched tensile bar AN2 (Figure 4) to establish the load–diametral contraction curve. During this kind of test, it is difficult to correctly measure the diameter contraction. In our study, the variation of the diameter has been measured using an image technique, which has the advantage of not requiring contact. An example of an image is shown in Figure 5. This technique follows the evolution of the diameter reduction exactly at the notch bottom and at the minimum diameter. In addition, the image analysis technique easily detects the point where crack initiation is localised. Figure 6 shows the evolution of the applied load as a function of diameter reduction \( \Delta d \) for the notched round tensile bar with 2 mm radius (AN2). At point ‘P’, there is a sharp change in the slope and a drastic fall in load, showing the initiation of failure process. This point gives the critical diameter reduction, \( \Delta d \), at crack initiation. The ductility \( \epsilon_f \) is defined as the average longitudinal strain at fracture

\[
\epsilon_f = 2 \ln \left( \frac{d_0}{d_0 - \Delta d} \right)
\]

\( d_0 = 6 \) mm is the initial diameter at the notch.

In our experimental test, \( \Delta d = 1.6 \) mm and so

\( \epsilon_f(\text{experiment}) = 0.62 \).

### FE analysis

Detailed elastic–plastic, axisymmetric FE damage analyses based on the GTN model, described in ‘Materials and experimental procedure’, were performed to simulate tensile tests of smooth and notched round bars using ABAQUS/Explicit. To incorporate a large geometry change effect, the large geometry change option was chosen. A velocity \( v(t) \) boundary condition was applied to the top of the FE model. In the quasi-static case, a 0.33 mm/min value was chosen. The resulting tensile load was determined from nodal forces. Axisymmetric conditions are imposed on the bottom and the left. Gauge length elongation was also monitored from the FE displacement results. The four-node solid element with reduced integration within ABAQUS/Explicit 6.5 (element type CAX4R) was used. It should be noted that the element size could be important in FE damage analysis. Convergence tests were performed in order to optimise the CPU time and consequently the number of elements and nodes in FE meshes are 660 elements/725 nodes to nodes, depending on a notch radius. Typical FE meshes, employed in this study are shown in Figure 7.

### Results and discussion

A parametric study of Gurson was carried out in order to show the effect of various parameters on the breakpoint of load–diametral contraction curve, \( F-\Delta d \), of an axisymmetric notched tensile bar AN2. The parameters for this point are essential to have the best fit between the numerical and experimental curves. The aim is to propose a methodology to determine the best set of parameters.

Table 1. Chemical composition of the 12NiCr6 steel.

<table>
<thead>
<tr>
<th>C</th>
<th>S</th>
<th>Si</th>
<th>Mn</th>
<th>Ni</th>
<th>Cr</th>
<th>Al</th>
</tr>
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<tr>
<td>0.12</td>
<td>0.007</td>
<td>0.032</td>
<td>0.6</td>
<td>1.6</td>
<td>0.85</td>
<td>0.176</td>
</tr>
</tbody>
</table>
Figure 8 presents the combined effect of the nucleation parameter $f_N$ and the critical parameter $f_c$ on the load–diametral contraction curve. We note that these two parameters strongly influence the increase of fracture strain $\sigma_f$. It believes inversely with the nucleation VVF of $f_N$. We also analyse the combined effect of each couple of parameters on the curve adjustment. This analysis shows that value quarters between $f_N$ and $f_c$ destabilise the numerical model's response, as shown in Figure 9. A gap must be maintained between the two parameters to avoid instability or divergence of the calculation process. These same remarks are valid between $f_0$ and $f_N$ and between $f_c$ and $f_F$.

Figure 10 summarises the precedent results. Shaded areas represent areas of divergence. This dictated the
choice of intervals of different parameters of the parametric study.

**Determination of $f_N$ and $f_c$ parameters**

The angular point load–diametral contraction curve, $F-\Delta d$, represents a sudden load drop. This point gives us the critical failure diameter. Corresponding deformation calculated from equation (8) represents the strain fracture or ductility. A parametric study (Figure 11) allows, for a given $f_0$, to plot a series of curves for different values of $f_c$.

Then, by drawing a family of curves $f_c(\epsilon_f)$ for varying $f_0$, we can quantify the effect of $f_0$ on $\epsilon_f$. (Figure 12) We note that for the materials with low initial porosity, as 12NiCr6 steel, the effect of this parameter is not important.

From this observation, determination method can be applied from the $f_c(\epsilon_f)$ curve for the value $f_0 = 1e-5$, representing the initial porosity 12NiCr6 steel.

For determination of $f_c$ value, experimental result of fracture strain $\epsilon_f(\text{experiment})$ was used, at which sudden drop of force occurs, which is caused by coalescence of voids in the material (Figure 8). The value obtained is $f_c = 0.06$ (Figure 13).

The term $f_N$ is determined by delaying experimental value of fracture strain on the $f_N(\epsilon_f)$ curve for $f_c = 0$.
06 curve. From this graphical construction, we deduce the value of $f_N = 0.0039$ (Figure 13).

**Effect of $f_N$ and $f_c$ parameters on the VVF variation**

Figure 14(a) shows the evolution of the VVF or porosity in the centre of the specimen. It is strongly influenced by the volume fraction in the nucleation $f_N$. For a low level of nucleation, the final volume fraction, $f_F$, i.e. the failure initiation reaches the high rates of plastic strain. However, for higher values of $f_N$, a smaller plastic strain is sufficient.

Figure 14(b) shows that the critical volume fraction, $f_c$, especially affects the coalescence acceleration. For low values of $f_c$, the final volume fraction is reached faster.

**Effect of $f_N$ and $f_c$ parameters on the equivalent plastic strain**

Figures 15 and 16 show the effect of $f_N$ on the changes in the critical VVF $f_c$ based on the equivalent plastic strain in the specimen centre. We also note that for low initial porosities the critical volume fraction $f_c$ varies a little according to the equivalent plastic deformation. (Figure 17).

**Visualisation of the crack propagation using GTN model**

Figure 18 shows the area where the final volume fraction is reached at the striction line, thus we can materialise the failure area. These elements are completely losing their rigidity and volume fraction reaches its final value. The Gurson model crack propagation described in this way.

**Conclusion**

The difficulty in the application of the GTN model lies in the following points:

- the significant number of parameters for identification
- no unique set of the parameters allowing fitting of the numerical curve on the experimental one.
In this article, we proposed a method which consists to lie the parameters of nucleation $f_N$ and of beginning of coalescence $f_c$ with physical parameters such as the initial VVF $f_0$ and the strain at failure $\varepsilon_f$. This method makes it possible to solve the unique problem of the parameter set.

This article presents a phenomenological model of ductile fracture for the 12NiCr6 steel using the GTN model. Experimental tests and FE damage simulations using the GTN model are performed for smooth and notched tensile specimens from which the parameters in the GTN model are calibrated.
An approach to parameter identification of the GTN model was presented using force–diametral contraction curves of axisymmetric notched specimen. Validity of the proposed parameters is checked by comparing the simulated results with the experimental ones from the notched bar. The dependence between the parameters of the GTN model and strain at failure $\varepsilon_f$ was highlighted by studying the variations of

![Figure 12. Identification of $f_c$.](image1)

![Figure 13. Nucleation VVF vs. fracture strain $f_c$ effect.](image2)

![Figure 14. VVF variation in the specimen centre: (a) $f_N$ effect; (b) $f_c$ effect.](image3)
Figure 15. Effect of $f_N$ on the $f_c$ vs. equivalent plastic strain.

Figure 16. Effect of $f_c$ on the $f_N$ vs. equivalent plastic strain.

Figure 17. Effect of $f_0$ on the $f_c$ vs. equivalent plastic strain.
parameters \( f_c \) and \( f_N \) depending on strain at failure \( \varepsilon_f \). From the graphs \( f_N(\varepsilon_f) \) and \( f_c(\varepsilon_f) \), we were able to identify the parameters of the critical volume fraction, \( f_c \), and the nucleation volume fraction \( f_N \). This approach allowed us to determine the critical volume fraction and nucleation for steel 12NiCr6, which are \( f_c = 0.06 \) and \( f_N = 0.004 \). The influence of triaxiality on the parameters is a very important aspect of the identification problem. This study was conducted on the notched tensile bar AE2, so for one triaxiality. A similar study on other specimens with different notch radii, i.e. different triaxialities is necessary. Influence of the volume rate of inclusions which characterises the initial porosity depends on the type of alloy to be considered. To complete the study, a validation on cracked specimens is necessary.

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