Computation of the net emission coefficient with the overlapping lines consideration on the CH$_4$-Ar plasma discharge

R.Benallal$^{1,2,a}$ and B.Liani$^{2,b}$

$^1$ Sciences & Engineering Sciences Faculty, University of Chlef 02000, Algeria
$^2$Theoretical physics Laboratory, Physics Department of University Aboubekr Belkaïd Tlemcen 13000, Algeria

$^a$rafik.benallal@gmail.com , $^b$b_liani2000@yahoo.fr

Keywords: Thermal plasma radiation. Net emission coefficient. Absorption coefficient

Abstract. Net emission coefficients of radiation were calculated for isothermal plasma of CH$_4$–Ar as a function of the plasma temperature 5,000–30,000 K and the arc radius (0–1 cm) at various plasma pressures. Calculations take into account continuum and line radiations. The line shapes in our calculations are given by convolution of Doppler and Lorentz profiles, resulting in a simplified Voigt profile. The overlapping lines have also been taken into account. In the case of hydrogen profile, we used the Vidal tables, and the four first Lyman lines and the four first Balmer lines have been considered in this work. Values of net emission coefficients calculated for various spectral regions were compared with others author’s works.

Introduction

In some applications of thermal plasmas, the medium consists of a mixture of various gases. Modeling of such plasmas and calculation of the energy balance require a deep knowledge of the radiative transfer. Even then, there is one property which remains difficult to calculate, that is, the net emission coefficient (NEC) which corresponds to the locally radiated power and takes auto-absorption of the radiation into account. This property, it is qualitatively well established but it is very difficult to quantify. The statistical equilibrium laws are used, but the Planck’s law is not valid; plasma does not behave as a black body and any numerical arcs modelling must take into account the radiative transfer phenomena. In the present study, we report the calculation of NEC defined by Lowke [1], in which it is assumed that the plasma is isothermal at temperature $T$. In spite of the simplification, the values of NEC have been calculated by overlapping lines consideration over the temperature and pressure ranges of thermal plasma.

Method of calculation

The general study of radiation emitted by plasma is composed of two parts. The first part describes the different mechanisms responsible for the radiation emission, while the expression for the spectral emission coefficient is given in literature [2,3]. The total radiation results from the superposition of the continuum and the line spectra.

The continuum spectrum is due to a number of radiative processes as follows [3]:

Radiative attachment, radiative recombination and Bremsstrahlung effect.

The line spectrum is produced when an electron in a given atom can jump from one energy level $E_m$ to another energy level $E_n$. Some lines are supposed to be non-self-absorbed, they cross the medium without being absorbed; and the other lines are strongly self-absorbed. They are all resonance lines or lines whose lower level is the first excited level of atoms or ions.

The second part solves the equation of radiative transfer and determines the net emission escaping from the plasma, which is not in complete thermodynamic equilibrium (LTE). A large amount of emitted radiation escapes from the medium without being absorbed and the Planck law cannot be applied to it, it is replaced by Kirchhoff's law:
\[ \varepsilon_\nu = k'_\nu B_\nu \]  

To calculate the radiation which escapes from the plasma, we must solve the equation of radiative transfer written in the form of [4] :

\[ \varepsilon_{N\nu} = k'_\nu (B_\nu - J_\nu) \]  

With

\[ J_\nu = B_\nu [1 - G_1(k'_\nu R)] \]  

Where \( G_1(k'_\nu R) \) is given [4] :

\[ G_1(k'_\nu R) = \frac{\pi}{2} \sin \theta \exp \left( \frac{k'_\nu R}{\sin \theta} \right) d\theta \]  

\( k'_\nu \) is the absorption coefficient corrected for the induced emission.

\[ k'_\nu = k_\nu \left[ 1 - \exp \left( -\frac{h\nu}{kT} \right) \right] \]  

\( B_\nu \) is the blackbody radiation density; \( J_\nu \) is the average radiation intensity; \( \varepsilon_\nu \) is the spontaneous emission monochromatic per unit of volume, solid angle, and time, also known as a function source or a spectral emission coefficient. The emission coefficient depends on the medium geometry and the radial temperature profile. The central region of the arc is the hottest part and therefore is the most emissive. The emitted radiation in this region will be strongly re-absorbed when it crosses the plasma. In the case of an isothermal plasma with a simplified cylindrical geometry, it is possible to write, in anisotropic medium, NEC in the form of :

\[ \varepsilon_{N\nu} = k'_\nu B_\nu G_1(k'_\nu R) \]  

where \( G_1 \) is a function accounting for the cylindrical geometry of the plasma and \( R \) is its radius (or the thickness of the medium). Libermann and Lowke [5] have shown that one can replace the isothermal cylinder by an isothermal sphere. Under these conditions, one has:

\[ G_1(k'_\nu R) = \exp \left( -k'_\nu R \right) \]  

and the form developed out of Eq. (6) is written as follows:

\[ \varepsilon_{N\nu}(T) = \int_0^\infty B(\nu,T)[k_c + k_0P(\nu)] \left[ 1 - \exp \left( -\frac{h\nu}{kT} \right) \right] \exp \left\{ -(k_0 + k_0P(\nu)) \left[ 1 - \exp \left( -\frac{h\nu}{kT} \right) \right] \right\} d\nu \]  

where \( k_c \) is the absorption coefficient of the continuum, \( k_0 \) is the absorption coefficient on the medium line and \( P(\nu) \) is the line profile. To calculate NEC one must know its profile, which is related to the broadening phenomena [3]. At high density, the lines are broadened by the thermal motion of emitting atoms known as the Doppler effect, this profile is called a Gaussian profile. The emitting atom can be subjected to the interactions of the surrounding particles, which causes a pressure effect (Stark, Van der Waals, and resonance broadening); under these conditions, one obtains a Lorentz profile. When the Stark profile is Lorentzian, the resultant profile, taking the Doppler effects into account, is a Voigt one. This profile is present in the literature [6]. The line profile of hydrogen does not have the same form (Voigt profile). The profile is close to a Holtzmark one with a very complicated expression. In our calculation, we use the values tabulated by Vidal [7]. All the theoretical calculations involving transport and emission coefficients etc require
knowledge of the plasma chemical composition. When the plasma is in LTE, quantitative relationships are established between substances taking part in the plasma composition [8]. Under these conditions, a calculation of the densities for different species is possible with an application of the following equilibrium laws: Saha law; Guldberg Waage law; Dalton law; Electrical neutrality; Conservation law of matter.

**Results and discussion**

This calculation was carried out in the case of isothermal plasma in cylindrical geometry, by considering the overlapping in the radiative transfer calculation. This approximation is valid in the central region of the arc, where the variations in temperature are weak. We could obtain a real absorption spectrum from 32 to 4500 nm range with a 10⁻³ nm wave-length step, which enabled us to calculate, using language FORTRAN, the net emission coefficient used to represent the radiative losses energy in the arcs modeling. Fig.1 represents the total radiation emitted by the plasma. This radiation is partially re-absorbed with its crossing; the plasma absorption increases with optical thickness and therefore with radius. However, Fig. 1 shows that a great part of the radiation, around 0.90, is absorbed in the first crossed millimeter, especially when the temperature is below 15,000 K. This corresponds to the resonance lines (the self-absorbed lines). Especially when the plasma is optically thin (R=0 mm), NEC is roughly proportional to the pressure because of its dependence on the plasma composition which itself depends on the pressure through the ideal gas law (Dalton's law). Fig. 2 illustrates this relation which exists between the total radiation and pressure. Two comparisons are made which take absorption into account. The first is between the results of the total radiation using a “line by line” method from Essoltani [9] and ours results and is illustrated in Fig.3. In order to show the validity of our results, Fig.4 shows the second comparison between our calculated values carried out for NEC related to lines where the wavelength is beyond 200 nm and those from measurements by Evans [10]. This result is quite consistent.

**Fig.1.** NEC for different radius of methane-argon plasma with P=1 atm

**Fig.2.** Total radiation of plasma for various pressure at R=0 mm.
Summary

Conclusions can be drawn as follows. a. the contribution of resonance lines plays a significant part in the emitted radiation. These lines are strongly absorbed in the first crossed millimeter. b. the NEC is roughly proportional to the pressure. c. the first comparison between Essoltani and ours results shows that when the overlapping of the lines is neglected, it overestimates the NEC. This is expected since the absorption coefficient is smaller. d. the NEC calculation at atmospheric pressure for pure argon agrees fairly well with the measurement by Evans.

References


Fig.3. Comparison of Essoltani and ours results for a pure plasma argon at R=1.0 mm with P=1 atm

Fig.4. Comparison of ours results with the Evans measurement for a pure plasma argon